Computation of Simplest Normal Forms of Differential Equations

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ABSTRACT
Normal form theory plays an important role in the study of nonlinear differential equations related to complex behavior patterns. The basic idea of the normal form theory is employing successive, almost linear transformations to find a new system called normal form, which is usually considered as a simple form. However, it has been found that a form generated by the normal form theory is not the simplest form. We propose a new concept “simplest normal form” and show how to compute the simplest normal for given differential equations. In particular, for Hopf bifurcation, suppose a conventional normal form derived from the normal form theorem is given by (using polar coordinates):

\[ \dot{r} = a_1 r^3 + a_2 r^5 + a_3 r^7 + \cdots + a_n r^{2n+1} + \cdots \]
\[ \dot{\theta} = 1 + b_1 r^2 + b_2 r^4 + b_3 r^6 + \cdots + b_n r^{2n} + \cdots \]

Then, the simplest normal form of Hopf bifurcation up to any order is:

\[ \dot{r} = a_1 r^3 + a_2 r^5 \]
\[ \dot{\theta} = 1 + b_1 r^2 \]

if \( a_1 \neq 0 \). Similar results have also been obtained for generalized Hopf bifurcations (when \( a_1 = 0 \)). The simplest normal forms may have influence on some fundamental theory of dynamic systems. Physical examples and symbolic computation using Maple will also be presented in the talk.